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The D2 SUSY zoo

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Abstract: We present new supersymmetric solutions of the Dirac-Born-Infeld equations for time-independent D2-branes, including a 1/2 supersymmetric 'dyonic' D2-brane and various 1/4 supersymmetric configurations that include 'twisted' supertubes, superfunnels with arbitrary planar cross-section, asymptotically planar D2-branes, and non-singular intersections of 'magnetic' D2-branes. Our analysis is exhaustive for D2-branes in three space dimensions.

Keywords: D-branes, M-Theory.

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1. Introduction

The low-energy dynamics of D-branes is governed by the Dirac-Born-Infeld (DBI) action, which generalizes the Dirac brane action to include worldvolume Born-Infeld (BI) electric and magnetic fields. These fields allow many more stable configurations than would otherwise be possible since they can support an otherwise unstable geometry against collapse. In many cases the stability can be understood as being a consequence of partial preservation of the supersymmetry of the string theory vacuum. Although many such 'supersymmetric' solutions of the DBI equations have been found, and their physical implications explored, there has not yet been any systematic attempt to find *all* supersymmetric solutions, in contrast to the situation for branes without worldvolume fields for which the mathematics of calibrations allows a complete classification for branes *in vacuo* [1, 2].

Here we initiate a program to classify all time-independent supersymmetric solutions of the DBI equations for the simplest case of a super D2-brane in the 10-dimensional Minkowski vacuum of IIA superstring theory (and all fractions of supersymmetry will refer to fractions of the 32 supersymmetries preserved by this vacuum). All such solutions have an M-theory interpretation as supersymmetric (although not necessarily time-independent) M2-branes [3 – 6] so it might seem that this case is too simple to yield anything new. However, the identification of the 'extra' space coordinate needed for S^1-compactification to the IIA Minkowski vacuum introduces some subtleties even for the 1/2 supersymmetric planar D2-branes. Although all such solutions descend from a planar M2-brane, the 1/2-supersymmetric D2-branes can be classified according to whether they are 'vacuum', 'electric', 'magnetic' or 'dyonic', and the 'dyonic' case is, (as far as we are aware) a new solution of the DBI equations.

This classification is reminiscent of the classification of intersecting planar branes in relative motion according to whether the intersection velocity is subluminal, superluminal or equal to the velocity of light [7]. This similarity is not a coincidence. The identification of the coordinate of the M-theory circle breaks the boost invariance in the 'extra' direction, leading to a foliation of the 11-dimensional spacetime by a family of timelike hypersurfaces that are at rest, in an *absolute* sense. One may now consider the motion of any object in M-theory, such as an M2-brane, with respect to any one of these rest-frame hypersurfaces, which we call 'ether-9-branes', or 'E9-branes', not only because they are 'etherial' (in the sense of having no local physical properties) but also because, collectively, they play a role analogous to that of the ether in pre-relativistic physics. Our classification of 1/2 supersymmetric D2-branes corresponds to the classification of M2-E9 intersections according to the same scheme as in [7], and the 'null' intersection yields the dyonic 1/2 supersymmetric D2-brane.

The possibilities for time-independent supersymmetric D2-branes preserving less than 1/2 supersymmetry are, of course, much more numerous. Our analysis is exhaustive only for D2-branes in a 3-dimensional subspace of the 9-dimensional Euclidean space, for which we find that all supersymmetric D2-branes are either 1/2 or 1/4 supersymmetric. Some of the 1/4 supersymmetric solutions are already known. An example is the original supertube [8], which is a tubular D2-brane supported by the angular momentum in the BI fields. Although it is time-independent, it should be considered stationary rather than static because of the non-zero angular momentum; this feature is explicit in its IIB superstring-theory dual manifestation as a "superhelix" [9] and in its M-theory manifestation as an "M-ribbon" [10]. We find a new tubular solution that we call a 'twisted' supertube because the electric field lines twist around the tube. A twisted supertube is actually just a supertube that has been boosted along its axis

The original D2-brane supertube was assumed to have a planar and circular cross-section, but it was soon realized that other cross-section shapes are possible [11 – 13]. Indeed, it was shown in [14] that 1/4 supersymmetry allows an arbitrary cross-sectional curve (which need not even be closed, although the description as a 'tube' becomes inappropriate

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if the curve is infinite). This surprising feature is readily understood from the TST-dual manifestation of the supertube as a "supercurve" [15], which is a IIA string in the T-dual direction carrying a wave with an arbitrary profile. Given this result, it is natural to wonder whether there exist supersymmetric 'tubular' D2-branes for which the scale of the cross-section varies along the length of the tube. This possibility was considered in [8], where it was concluded that any such configuration would be equivalent to the circularly-symmetric "dyonic BIon" of [16], which was recovered as a 'tubular' configuration with a circular cross-section that varies exponentially along the tube; from this perspective the term "superfunnel" seems more appropriate. However, the circularly-symmetric superfunnel is found by choosing a *particular* solution of the two-dimensional Laplace equation. Here we exhibit solutions that yield superfunnels with an *arbitrary* (planar) cross-section. Other solutions of the two-dimensional Laplace equation yield other 1/4 supersymmetric "supershapes" including asymptotically-planar D2-branes.

We do not attempt a systematic study of D2-brane geometries in Euclidean spaces of more than three dimensions, but we partially analyze the conditions for 1/4 supersymmetry. One motivation for this partial analysis is that the generic cross section for a supertube in the 9-dimensional space of the 10-dimensional Minkowski IIA vacuum is known to be an arbitrary curve in the 8-dimensional space transverse to the supertube 'axis' [14] and we would like to know how this result generalizes. Not surprisingly, we find the same result for the twisted supertubes. In contrast, we find that the cross-section of a superfunnel is necessarily planar. Finally, we present a new 1/4 supersymmetric solution for a D2-brane in a 4-dimensional space that can be interpreted as a (non-singular) intersection of two, asymptotically planar, 'magnetic' D2-branes.

We will begin with a summary of the DBI action for D2-branes, the supersymmetry preservation condition, and the relation to M2-brane configurations, thereby collecting together many of the basic formulas that we will need for the subsequent tour of the "D2 Susy Zoo".

2. Preliminaries

We choose cartesian coordinates for the 10-dimensional Minkowski metric, such that

$$ds_{10}^2 = -dT^2 + \sum_{I=1}^{9}(dX^I)^2 \,. \tag{2.1}$$

Let ξ^μ ($\mu = 0,1,2$) be the D2-brane's worldvolume coordinates. The induced worldvolume metric is then

$$ds_3^2 = g_{\mu\nu}\, d\xi^\mu d\xi^\nu \,, \tag{2.2}$$

where

$$g_{\mu\nu} = -\partial_\mu T \partial_\nu T + \sum_I \partial_\mu X^I \partial_\nu X^I \,. \tag{2.3}$$

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The low-energy dynamics for a D2-brane of unit surface tension is governed by the Dirac-Born-Infeld (DBI) action

$$I = -\int d^3\xi\, \Delta\,, \qquad \Delta \equiv \sqrt{-\det(g+F)} \tag{2.4}$$

where F is the BI 2-form field strength subject to the Bianchi identity $dF = 0$. Setting $\xi^\mu = (t, \sigma^i)$ $(i = 1, 2)$, we can write F as

$$F = E_i\, dt \wedge d\sigma^i + \mathcal{B}\, d\sigma^1 \wedge d\sigma^2\,, \tag{2.5}$$

where E_i is the BI electric field, and $\mathcal{B}$ the BI magnetic field density. The Bianchi identity is

$$\partial_t \mathcal{B} = \varepsilon^{ij} \partial_i E_j\,. \tag{2.6}$$

Similarly, we can now write the induced metric as

$$ds^2 = g_{00}\, dt^2 + 2g_{0i}\, dt d\xi^i + h_{ij}\, d\sigma^i d\sigma^j \tag{2.7}$$

so that $h_{ij} = g_{ij}$. This allows us to define h^{ij} as the inverse to h_{ij} (with g^{ij} being the space components of the inverse to $g_{\mu\nu}$). After some calculation, one finds that

$$\Delta^2 = -\det g - \det h\, h^{ij} E_i E_j - g_{00}\, \mathcal{B}^2 - 2\mathcal{B}\, \varepsilon^{ij} E_i\, g_{0j}\,. \tag{2.8}$$

2.1 Supersymmetry preservation

Let (Γ_T, Γ_I) be the (constant) 32×32 spacetime Dirac matrices, which we may choose to be real. These matrices act on real $SO(1,9)$ spinors ϵ that we may decompose as

$$\epsilon = \epsilon_+ + \epsilon_-\,, \qquad \Gamma_\natural\, \epsilon_\pm = \pm \epsilon_\pm \tag{2.9}$$

where

$$\Gamma_\natural \equiv \Gamma_T \Gamma_1 \cdots \Gamma_9 \tag{2.10}$$

is the 10-dimensional (constant) chirality matrix. The spacetime Dirac matrices induce reducible (32×32), and ξ-dependent, worldvolume Dirac matrices γ_μ satisfying

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}\,. \tag{2.11}$$

We define $\gamma^\mu = g^{\mu\nu}\gamma_\nu$, and similarly for the antisymmetrized products of worldvolume Dirac matrices; for example

$$\gamma^{\mu\nu} = g^{\mu\rho} g^{\nu\lambda} \gamma_{\rho\lambda}\,, \qquad \left(\gamma_{\mu\nu} \equiv \gamma_{[\mu}\gamma_{\nu]}\right)\,. \tag{2.12}$$

The number of supersymmetries preserved by a given D2-brane configuration is the dimension of the space of solutions for *covariantly-constant* spinors ϵ of the equation

$$\Gamma \epsilon = \epsilon \tag{2.13}$$

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where Γ is the 'kappa-symmetry' matrix [6]

$$\Gamma = \frac{1}{\Delta}\left[\gamma_{012} + \varepsilon^{ij} E_i \gamma_j \Gamma_\natural + \mathcal{B}\gamma_0\Gamma_\natural\right] . \tag{2.14}$$

In the cartesian coordinates used here, covariant constancy implies constancy, so ϵ must actually be a *constant* spinor. Note that Γ is traceless and satisfies the identity

$$\Gamma^2 \equiv 1 . \tag{2.15}$$

which implies preservation of 1/2 supersymmetry *locally.* However Γ is a function of position on the worldvolume, generically, so the fraction of supersymmetry preserved will generally be less than 1/2. In fact, generically there will be no non-zero solutions to (2.13) so D2-brane configurations preserving any non-zero fraction of supersymmetry must be special. Finally, note that

$$\{\Gamma, \Gamma_\natural\} = 0 , \tag{2.16}$$

which implies that all supersymmetries are broken by a restriction to chiral 10-dimensional spinors, as expected because there is no supersymmetric membrane solution of any minimal 10-dimensional supergravity theory.

2.2 Stationary D2-branes

Here we are interested in *stationary* (time-independent) supersymmetric D2-brane configurations, so it is convenient to fix the time-reparametrization invariance by the partial gauge choice

$$T = t . \tag{2.17}$$

We now have a static worldvolume metric with $g_{00} = -1$ and $g_{0i} = 0$:

$$ds_3^2 = -dt^2 + d\sigma^i d\sigma^j\, h_{ij} , \tag{2.18}$$

where

$$h_{ij} = \sum_{I=1}^{9} \partial_i X^I \partial_j X^I , \tag{2.19}$$

and hence

$$\det h = \sum_{I>J} \left(\boldsymbol{\nabla} X^I \times \boldsymbol{\nabla} X^J\right)\left(\boldsymbol{\nabla} X^I \times \boldsymbol{\nabla} X^J\right) , \tag{2.20}$$

where we use the standard 2D vector calculus notation. We now have

$$\Delta^2 = (\det h)\left(1 - h^{ij} E_i E_j\right) + \mathcal{B}^2 , \tag{2.21}$$

where, since E_i and $\mathcal{B}$ are now assumed to be t-independent, the Bianchi identity $dF = 0$ reduces, in 2D vector calculus notation, to

$$\boldsymbol{\nabla} \times \mathbf{E} = 0 . \tag{2.22}$$

The induced worldvolume Dirac matrices are

$$\gamma_0 = \Gamma_T , \qquad \gamma_i = \partial_i X^I \Gamma_I \tag{2.23}$$

so that $\gamma_{012} = \Gamma_T \gamma_{12}$, where

$$\gamma_{12} = \frac{1}{2} \left(\boldsymbol{\nabla} X^I \times \boldsymbol{\nabla} X^J \right) \Gamma_{IJ} \,. \tag{2.24}$$

The kappa-symmetry matrix Γ is

$$\Gamma = \frac{1}{\Delta} \left[\frac{1}{2} \Gamma_T \left(\boldsymbol{\nabla} X^I \times \boldsymbol{\nabla} X^J \right) \Gamma_{IJ} + \varepsilon^{ij} E_i \partial_j X^I \Gamma_{I\natural} + \mathcal{B}\, \Gamma_{T\natural} \right] . \tag{2.25}$$

It is important to appreciate that a stationary supersymmetric D2-brane will satisfy the DBI equations if and only if the Gauss law constraint,

$$\partial_i \mathcal{D}^i = 0 \,, \tag{2.26}$$

is satisfied, where the electric 'displacement' field density is

$$\mathcal{D}^i \equiv -\frac{\delta \Delta}{\delta E_i} = \Delta^{-1} \det h \, h^{ij} E_j \,. \tag{2.27}$$

This follows from consideration of the Hamiltonian formulation; we will not need this formalism here but we record that the Hamiltonian density $\mathcal{H}$ for a stationary D2-brane in the gauge $T = t$ is given by

$$\mathcal{H}^2 = \det h + \mathcal{B}^2 + h_{ij} \mathcal{D}^i \mathcal{D}^j \left[1 + \mathcal{B}^2 / \det h \right] . \tag{2.28}$$

2.3 Lift to M-theory

The D2-brane has an M-theory interpretation as the 11-dimensional supermembrane, or M2-brane [3, 4]. Let $X^\natural$ be the 10th cartesian space coordinate, which becomes the angular coordinate of the M-theory circle after periodic identification. The unit tension M2-brane has the action[1]

$$I_{M2} = -\int d^3\xi \det \left(g^{(M2)} \right) . \tag{2.29}$$

The induced worldvolume metric is

$$g^{(M2)}_{\mu\nu} = g_{\mu\nu} + \partial_\mu X^\natural \partial_\nu X^\natural \tag{2.30}$$

where $g_{\mu\nu}$ is the induced metric of (2.3). Following the steps spelled out in detail in [6], one finds that the derivatives of the M2 worldvolume field $X^\natural(\xi)$ are related to the BI fields of the D2-brane as follows:

$$\partial_\mu X^\natural = (\pm) \frac{1}{2\Delta} \, g_{\mu\nu} \varepsilon^{\nu\lambda\rho} F_{\lambda\rho} \,, \tag{2.31}$$

where $(\pm)$ denotes an appropriate sign corresponding to the orientation of the D2 embedding.

Let us now specialize to the case of a stationary brane. In the gauge $T = t$, one has

$$\dot{X}^\natural = -(\pm) \Delta^{-1} \mathcal{B} \,, \qquad \partial_i X^\natural = -(\pm) \Delta^{-1} h_{ij} \varepsilon^{jk} E_k \,. \tag{2.32}$$

[1] We omit the fermions, as they are irrelevant for the present purposes.

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Equivalently,

$$\mathcal{B} = -(\pm)\dot{X}^{\natural}\,\Delta\,, \qquad E_i = (\pm)h_{ij}\,\varepsilon^{jk}\partial_k X^{\natural}\,(\Delta/\det h)\,. \tag{2.33}$$

Using these expressions in Δ and solving the resulting equation for Δ, one finds that

$$\Delta = \frac{\sqrt{\det h}}{\sqrt{1-\left(\dot{X}^{\natural}\right)^2 + h^{ij}\partial_i X^{\natural}\partial_j X^{\natural}}}\,. \tag{2.34}$$

Note that the quantity

$$-\frac{1}{2}F^{\mu\nu}F_{\mu\nu} = h^{ij}E_iE_j - \left(\mathcal{B}^2/\det h\right) = \left[h^{ij}\partial_i X^{\natural}\partial_j X^{\natural} - \left(\dot{X}^{\natural}\right)^2\right]\left(\Delta^2/\det h\right) \tag{2.35}$$

is invariant with respect to the $Sl(2;\mathbb{R})$ worldvolume Lorentz group.

3. One-half supersymmetry

Before considering configurations preserving less than 1/2 supersymmetry, we consider the condition for 1/2 supersymmetry, which is

$$\left[\frac{1}{2}\Gamma_T\left(\boldsymbol{\nabla}X^I\times\boldsymbol{\nabla}X^J\right)\Gamma_{IJ} + \varepsilon^{ij}E_i\partial_jX^I\Gamma_{I\natural} + \mathcal{B}\,\Gamma_{T\natural} - \Delta\right]\epsilon = 0\,. \tag{3.1}$$

As ϵ is constant, this condition will imply preservation of 1/2 supersymmetry only if all terms are proportional to Δ. In particular we require

$$\boldsymbol{\nabla}X^I\times\boldsymbol{\nabla}X^J = \Omega^{IJ}\,\Delta \tag{3.2}$$

for some constant antisymmetric 9×9 matrix Ω^{IJ}. Fixing the worldspace diffeomorphisms by choosing $X^1=\sigma^1, X^2=\sigma^2$ we learn that Δ is constant in this gauge, in which case all space components X^I must be linear in the worldspace coordinates σ^i in order that $\boldsymbol{\nabla}X^I\times\boldsymbol{\nabla}X^J$ be constant for all (I,J). This implies that the brane geometry is a plane, with the constants $\boldsymbol{\nabla}X^I\times\boldsymbol{\nabla}X^J$ being proportional to the projections of an area element of the brane onto the $I-J$ plane. Clearly, we may orient the planar brane such that the only non-zero entries of Ω are $\Omega^{12}=-\Omega^{21}=1/\Delta$. We then have $h_{ij}=\delta_{ij}$, and hence

$$\Delta^2 = 1+\mathcal{B}^2-|\mathbf{E}|^2\,. \tag{3.3}$$

3.1 Classification

Now that we know that Δ is constant, the requirement that all coefficients in (3.1) be proportional to Δ implies that

$$\mathbf{E} = \mathbf{n}\,E\,, \qquad \mathcal{B} = B\,, \tag{3.4}$$

for fixed unit 2-vector $\mathbf{n}$ and *constants* E and B. The supersymmetry preservation condition now reduces to

$$\left[\Gamma_T\Gamma_{12} + \varepsilon^{ij}E\,n_i\Gamma_{j\natural} + B\Gamma_{T\natural} - \Delta\right]\epsilon = 0\,, \tag{3.5}$$

where

$$\Delta = \sqrt{1 - E^2 + B^2}\,. \tag{3.6}$$

This condition can be satisfied for any constants (E, B) provided that $E < \sqrt{1+B^2}$, and so all such planar D2-brane configurations preserve 1/2 supersymmetry.

The energy density is

$$\mathcal{H} = \frac{1 + B^2}{\sqrt{1 + B^2 - E^2}} \geq 1\,, \tag{3.7}$$

with equality for the 'vacuum' D2-brane with $E = B = 0$, which is the only 1/2 supersymmetric configuration that is invariant under the SO(1, 2) worldvolume Lorenz group. All other 1/2 supersymmetric static D2-branes break the worldvolume Lorentz invariance, and as $E^2 - B^2$ is a Lorentz invariant there are three cases to consider:

- $E^2 - B^2 < 0$. In this case we may boost to a frame in which $E = 0$. In this frame the non-zero magnetic density breaks SO(1, 2) to SO(2), the worldspace rotation group. Because the boost invariance is broken, a boost generates a new 1/2 supersymmetric D2-brane with electric as well as magnetic BI field.

- $E^2 - B^2 > 0$. In this case we may boost to a frame in which $B = 0$. The non-zero electric field in this frame, which is constrained by $|\mathbf{E}| < 1$, breaks SO(1, 2) to SO(1, 1), which is the group of boosts in the direction of the electric field. A boost in the orthogonal direction generates a new 1/2 supersymmetric D2-brane with a magnetic field.

- $E = B \neq 0$. This case is intrinsically 'dyonic' in the sense that there is no frame for which either the electric or the magnetic field is zero. We now have $\Delta = 1$ and

$$\Gamma = \Gamma_T \Gamma_{12} + B\left(\Gamma_T + \Gamma_2\right)\Gamma_\natural\,. \tag{3.8}$$

We may summarize this state of affairs by saying that a non-vacuum 1/2 supersymmetric D2-brane is 'magnetic', 'electric', or 'dyonic' according to whether the 3-vector (B, E_1, E_2) is, respectively, timelike, spacelike or null.

3.2 M-theory and E-branes

Each of the possible 1/2 supersymmetric D2-branes must lift to a planar M2-brane in the $\mathbb{E}^{(1,9)} \times S^1$ vacuum of M-theory, and it is instructive to see how the distinction between magnetic, electric and dyonic D2-branes arises in this context. As in subsection 2.3, we let $X^\natural$ be the coordinate of the M-theory circle. We may assume that the plane of M2-brane is spanned by a vector along the 1-axis, say, and another vector in the $2-\natural$ plane. Taking into account that $(X^1, X^2) = (\sigma^1, \sigma^2)$, this implies that $X^\natural$ is linear in σ^2. Allowing, too, for a linear time-dependence of $X^\natural$, we have

$$\dot{X}^\natural = u\,, \qquad \partial_2 X^\natural = \tan\theta\,, \tag{3.9}$$

for constant u and constant angle θ, which is the angle that the M2-brane makes to the X^2-axis. From subsection 2.3, we read off the corresponding BI fields:

$$E = E_1 = \frac{\sin\theta}{\sqrt{1-v^2}}\,, \qquad B = -\frac{v}{\sqrt{1-v^2}}\,, \tag{3.10}$$

where

$$v = u\cos\theta \tag{3.11}$$

is the physical transverse velocity of the M2-brane. It follows that

$$E^2 - B^2 = \frac{\sin^2\theta\left(1 - v_{\rm int}^2\right)}{1-v^2}\,, \tag{3.12}$$

where we have defined

$$v_{\rm int} \equiv u/\tan\theta = v/\sin\theta\,. \tag{3.13}$$

We see from this result that the D2-brane will be 'electric' or 'magnetic' according to whether the velocity $v_{\rm int}$ is subluminal or superluminal, and the dyonic D2-brane corresponds to $v_{\rm int} = 1$. But what is the intrinsic significance of $v_{\rm int}$?

To answer this question, we begin by recalling a discussion of Bachas and Hull on intersecting branes [7]. If the velocity of the intersection is $v_{\rm int}$ then one must distinguish between the three cases $v_{\rm int} < 1$, $v_{\rm int} = 1$ and $v_{\rm int} > 1$. For $v_{\rm int} < 1$ one can boost to a frame in which the intersection is at rest, and in this frame one has static intersecting branes. For $v_{\rm int} > 1$ one can boost to a frame in which the branes are parallel but in relative motion. It seems as though a similar analysis should be applicable here but, if so, where is the 'other' brane with respect to which the M2-brane is in motion?

To answer *this* question, we recall that the identification of the $X^\natural$ coordinate breaks the 11-dimensional Lorentz invariance; in particular it breaks the invariance under boosts in the $X^\natural$ direction, thereby introducing globally-defined rest-frames for motion in this direction. Specifically, any hypersurface of constant $X^\natural$ is at rest, and can be viewed as a kind of 'etherial' 9-brane. Recalling that the hypothetical material defining the absolute rest frame in pre-relativistic physics was called the 'ether', we propose to call this an 'ether' 9-brane, or 'E9-brane'. An E9-brane has no local physical properties, but is nevertheless a convenient way of thinking about the globally-defined rest frames implied by the existence of the M-theory circle. It is convenient because we can now apply a Bachas-Hull-type argument to the M2-E9 intersection. The velocity of this intersection is precisely $v_{\rm int}$, so we get a classification of D2-branes according to whether $v_{\rm int}$ is less than, greater than or equal to the velocity of light. As we have just shown, this classification coincides with the 'intrinsic' classification into electric, magnetic and dyonic D2-branes.

4. D2 in 3D

In the following section, we will present a systematic analysis that uncovers all possible time-independent supersymmetric D2-branes for which the D2-brane geometry is a surface in Euclidean 3-space with coordinates (X^1, X^2, X^3). Many special cases can be found by non-systematic methods of course, and we are first going to present such an analysis, based on the original approach in [8]. We do this partly because we wish to correct an error in [8] which led to a puzzle that we resolve here, and also because it turns out that we do actually find all the 1/4 supersymmetric solutions this way.

Following [8], we introduce the cylindrical polar coordinates (R, Φ, X):

$$X^1 = R\cos\Phi\,, \qquad X^2 = R\sin\Phi\,, \qquad X^3 = Z\,. \tag{4.1}$$

For convenience of comparison with [8], we also relabel the worldspace coordinates

$$\sigma^1 = z\,, \qquad \sigma^2 = \varphi\,, \tag{4.2}$$

with φ an angular coordinate (such that $\varphi \sim \varphi + 2\pi$). This allows us to fix the worldspace parametrization invariance by the gauge choice

$$Z = z\,, \qquad \Phi = \varphi\,. \tag{4.3}$$

The induced worldspace line-element is

$$h_{ij}d\sigma^i d\sigma^j = \left(1 + R_z^2\right) dz^2 + 2R_zR_\varphi dz d\varphi + \left(R^2 + R_\varphi^2\right) d\varphi^2\,, \tag{4.4}$$

where $R_z = \partial_z R$ and $R_\varphi = \partial_\varphi R$. In this approach, the cross-section of the tube is assumed to be planar from the outset, but since R is, a priori, a function of both z and φ, allowance is made for a possible change of scale along the axis of the tube (which leads to what we will here call 'superfunnels') and an arbitrary planar shape (although only the circular supertube was actually found in [8]). We write the BI 2-form as

$$F = E_z\, dt \wedge dz + E_\varphi\, dt \wedge d\varphi + \mathcal{B}\, dz \wedge d\varphi\,. \tag{4.5}$$

Note the allowance of an electric field component E_φ around the tube as well as the component E_z along it. In this respect we have a generalization of the analysis of [8].

A calculation now yields

$$\Delta^2 = \left(R^2 + R_\varphi^2\right)\left(1 - E_z^2\right) + \mathcal{B}^2 + R^2R_z^2 - E_\varphi^2\left(1 + R_z^2\right) + 2R_zR_\varphi E_zE_\varphi\,, \tag{4.6}$$

from which we compute that

$$\begin{aligned} \mathcal{D}_z &= \Delta^{-1}\left[\left(R^2 + R_\varphi^2\right)E_z - R_zR_\varphi E_\varphi\right]\,, \\ \mathcal{D}_\varphi &= \Delta^{-1}\left[\left(1 + R_z^2\right)E_\varphi - R_zR_\varphi E_x\right]\,. \end{aligned} \tag{4.7}$$

The Hamiltonian density is

$$\mathcal{H} = \Xi^{-1}\sqrt{\left[\Xi^2 + |\mathcal{D}|^2\right]\left[\Xi^2 + \mathcal{B}^2\right]} \tag{4.8}$$

where

$$\begin{aligned} \Xi^2 &\equiv R^2\left(1 + R_z^2\right) + R_\varphi^2\,, \\ |\mathcal{D}|^2 &= \left(1 + R_z^2\right)\mathcal{D}_z^2 + 2R_zR_\varphi\mathcal{D}_z\mathcal{D}_\varphi + \left(R^2 + R_\varphi^2\right)\mathcal{D}_\varphi^2\,. \end{aligned} \tag{4.9}$$

The induced worldvolume Dirac matrices are

$$\gamma_0 = \Gamma_T\,, \qquad \gamma_z = \Gamma_Z + R_z\Gamma_R\,, \qquad \gamma_\varphi = R\Gamma_{\underline{\Phi}} + R_\varphi\Gamma_R \tag{4.10}$$

where $\Gamma_{\underline{\Phi}}$ is the Dirac matrix in the obvious orthonormal frame; it is constant and squares to the identity matrix. Following [8], we note that a covariantly constant spinor in the new cylindrical polar coordinates is not constant but instead takes the form

$$\epsilon = \exp\left(\frac{1}{2}\Phi\Gamma_{R\underline{\Phi}}\right)\epsilon_0\,. \tag{4.11}$$

Following the steps sketched in [8], and taking into account that $\Phi = \varphi$, we now find that the supersymmetry preservation condition can be put into the form

$$0=\left[RR_z\Gamma_{TR\underline{\Phi}}+\mathcal{B}\Gamma_{T\natural}-E_\varphi\Gamma_{Z\natural}-\Delta\right]\epsilon_0+\exp\left(-\varphi\Gamma_{R\underline{\Phi}}\right)\left[\gamma_\varphi\Gamma_\natural\left(\Gamma_{TZ\natural}+E_z\right)-E_\varphi R_z\Gamma_{R\natural}\right]\epsilon_0\,. \tag{4.12}$$

For $E_\varphi = 0$, this should reduce to the condition found in [8] but it does not quite do so. The first set of bracketed terms in eq. (20) of [8] erroneously includes an additional $R_zR_\varphi\Gamma_T$ term that led the authors to conclude that 1/4 supersymmetry requires either $R_z = 0$ or $R_\varphi = 0$. Since only such configurations were then considered, this error had no further consequences in [8] but we must now re-analyse the possibilities, which we do taking into account the additional possibility of non-zero E_φ.

If one assumes that the worldvolume fields are φ-independent, then, as argued in [8], the two square-bracketed terms on the right hand side of (4.12) must vanish independently. This requirement is not obviously necessary when the worldvolume fields are allowed to depend on φ but, as will be seen in the following section, relaxation of it does not yield any new solutions. We therefore proceed by assuming that each of the two bracketed terms is zero. Observing that the gamma-matrices in the first two terms of the second bracket square to the identity whereas the gamma-matrix in the last term squares to minus the identity, we deduce that the terms of the second bracket vanish if and only if

$$E_z = \pm 1\,, \qquad E_\varphi R_z = 0\,, \tag{4.13}$$

and

$$\Gamma_{TZ\natural}\,\epsilon = \mp\epsilon\,. \tag{4.14}$$

The supersymmetry preservation condition then becomes

$$\left[RR_z\,\Gamma_{TR\underline{\Phi}} + \mathcal{B}\,\Gamma_{T\natural} - E_\varphi\,\Gamma_{Z\natural} - \Delta\right]\epsilon_0 = 0 \tag{4.15}$$

where, now,

$$\Delta^2 = \mathcal{B}^2 + (RR_z)^2 - E_\varphi^2\,. \tag{4.16}$$

We see from (4.13) that there are two alternatives: either (i) $R_z = 0$ or (ii) $E_\varphi = 0$. We will consider them in turn.

4.1 Supertubes

For the choice

$$R_z = 0\,, \tag{4.17}$$

the supersymmetry preservation condition becomes

$$\left(\mathcal{B}\Gamma_{T\natural} + E_\varphi \Gamma_{Z\natural}\right)\epsilon = \sqrt{\mathcal{B}^2 - E_\varphi^2}\;\epsilon\,. \tag{4.18}$$

Since both $\Gamma_{T\natural}$ and $\Gamma_{Z\natural}$ commute with $\Gamma_{TZ\natural}$, this condition is compatible with (4.14), but the two together imply preservation of 1/4 supersymmetry only if

$$E_\varphi = \beta\mathcal{B}\,, \qquad \beta^2 < 1 \tag{4.19}$$

for some constant β. The Gauss law reduces to $\partial_z \mathcal{B} = 0$ in this case, consistent with invariance under translations along the Z-axis. The standard supertube, with an arbitrary planar cross-section, is found by setting $E_\varphi = 0$. Details of the $E_\varphi \neq 0$ case will be left until our later discussion allowing for a non-planar cross-section.

We now have a 1/4 supersymmetric tubular configuration determined by the arbitrary functions $R(\varphi)$ and $\mathcal{B}(\varphi)$. To facilitate comparison with the more general solutions that we present in section 7, we recall here that we fixed the worldspace diffeomorphism by setting $Z = z$ and $\Phi = \varphi$, so that the functions $X^1(\varphi)$ and $X^2(\varphi)$ are not independent because they are both determined by the function $R(\varphi)$. However, we could now suppose that φ is a function of some new angular variable ψ, in which case

$$\mathcal{B}\, dz \wedge d\varphi = B\, dz \wedge d\psi \tag{4.20}$$

where $B = \mathcal{B}(\partial\varphi/\partial\psi)$ is a *constant*. In this reparametrization of the solution, it is now the two functions $X^1(\psi)$ and $X^2(\psi)$ that are independent, and they determine an arbitrary curve in the X^1-X^2 plane.

4.2 Superfunnels

We now suppose that $R_z \neq 0$, in which case we must set

$$E_\varphi = 0\,. \tag{4.21}$$

The supersymmetry preservation condition is now

$$\left(RR_z\Gamma_{TR\underline{\Phi}} + \mathcal{B}\Gamma_{T\natural}\right)\epsilon = \sqrt{\mathcal{B}^2 + R^2R_z^2}\,\epsilon\,. \tag{4.22}$$

Since both $\Gamma_{TR\underline{\Phi}}$ and $\Gamma_{T\natural}$ commute with $\Gamma_{TZ\natural}$, this condition is compatible with (4.14), but the two together imply preservation of 1/4 supersymmetry only if

$$\mathcal{B} = B_0 R R_z \tag{4.23}$$

for some constant B_0. The Gauss law constraint in this case is

$$\partial_z\left[\frac{R}{R_z} + \frac{R_\varphi^2}{RR_z}\right] = \partial_\varphi^2 \log R\,. \tag{4.24}$$

To see the significance of this constraint, we switch independent variables from (z, φ) to $(\rho \equiv R, \varphi)$, taking z, which equals Z, as the dependent variable, and use the relations

$$(\partial_z)_\varphi = \frac{1}{(Z_\rho)_\varphi}\,(\partial_\rho)_\varphi\,, \qquad (\partial_\varphi)_z = (\partial_\varphi)_\rho - \frac{(Z_\varphi)_\rho}{(Z_\rho)_\varphi}\,(\partial_\rho)_\varphi\,. \tag{4.25}$$

We find that

$$R_z = 1/Z_\rho\,, \qquad R_\varphi = -Z_\varphi/Z_\rho\,, \tag{4.26}$$

where it should be clear which variables are being kept fixed from the convention that lower/upper case variables are independent/dependent. One then finds that (4.24) is equivalent to

$$\nabla^2 Z = 0\,. \tag{4.27}$$

In other words Z is a solution of the 2D Laplace equation. Note too that

$$F = \pm\, dt \wedge dz + \mathcal{B} dz \wedge d\varphi = \pm(\partial_{X^i} Z)\, dt \wedge dX^i + B_0 dX^1 \wedge X^2\,. \tag{4.28}$$

The general periodic solution of the Laplace equation is

$$Z = Z_0 + Q \log\rho + {\sum_{k\in\mathbb{Z}}}' \left[(c_k \cos(k\varphi) + \tilde{c}_k \sin(k\varphi))\right]\rho^k \tag{4.29}$$

for constants Z_0, Q and $(c_k, \tilde{c}_k)$, where the prime on the sum indicates that the $k = 0$ term is omitted. It was noted in [8] that the particular solution $Z = Z_0 + Q\log\rho$ is equivalent to the dyonic BIon of [16]. More generally,

$$\oint d\varphi\, Z \propto \log\rho\,. \tag{4.30}$$

for any solution of the Laplace equation with non-zero Q. This is the expected logarithmic bending of a D2-brane due to a charge Q, which can be interpreted as the charged endpoint of an infinite IIA string. Equivalently, we have a IIA string that has been 'blown-up' to a funnel-shaped D2-brane, with a planar cross-section of arbitrary shape that grows exponentially along the axis of the funnel, at least 'on average'. Since they preserve 1/4 supersymmetry we call them 'superfunnels'.

4.3 Other supershapes

When $Q = 0$, we have a new type of 1/4 supersymmetric D2-brane that is neither a supertube nor a superfunnel, arising from the multipole expansion of Z. A simple example, which might be called a "super-dipole" is

$$Z = \cos\varphi/\rho\,. \tag{4.31}$$

This has the feature that $Z \to 0$ as $\rho \to \infty$, so the brane geometry is asymptotically planar. We depict the shape of the surface in figure 1. One finds that $\mathcal{B} = B_0 R R_z = -B_0\rho^3/\cos\varphi$ for this configuration, which appears to blow up for $\varphi = \pi/2$ and as $\rho \to \infty$, but this is

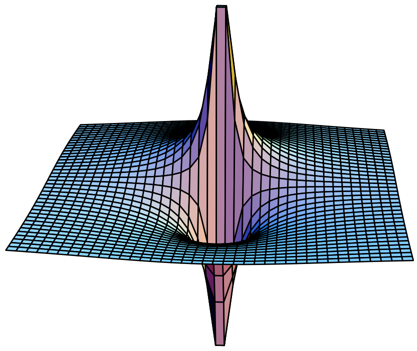

Figure 1: The simple example (4.31) that is asymptotically planar.

an artifact of the choice of coordinates. As we showed above, $\mathcal{B}$ is proportional to the area element and hence constant in local cartesian coordinates.

The projections on the spinor ϵ required by the 1/4 supersymmetry of all these $Q = 0$ solutions are exactly the same as for the superfunnel solutions, which suggests an interpretation as a IIA string that passes through a planar D2-brane. If the point of intersection of such a string is split, one has two string ends of opposite charge. Such a zero charge distribution would have multipole moments of all orders, so the pure dipole solution of (4.31) must correspond to a much smoother charge distribution of zero net charge, which is possibly why it does not have the appearance of a string intersecting a D2-brane.

Let us consider the M-theory lift of the "super-multipole" solutions. Taking into account a sign associated to the orientation of D2 embedding, one has

$$(\pm)\Delta = \sqrt{1+B_0^2}\, RR_z = \sqrt{1+B_0^2}\frac{\cos^2\varphi}{z^3}\,. \tag{4.32}$$

From (2.32) we see that the velocity in the $\natural$ direction is

$$\dot{X}^\natural = \frac{B_0}{\sqrt{1+B_0^2}}\,. \tag{4.33}$$

In fact, this result also holds for any of the superfunnel configurations, and it implies that both superfunnels and super-multipoles are moving uniformly with constant velocity along the M-theory circle.

Since $E_\varphi = 0$, the second equation of (2.32) becomes, for the configuration defined by (4.31),

$$\partial_z X^\natural = \frac{\tan\varphi}{\sqrt{1+B_0^2}}\,, \qquad \partial_\varphi X^\natural = \frac{z}{\sqrt{1+B_0^2\cos^2\varphi}}\,. \tag{4.34}$$

These equations can be integrated straightforwardly:

$$X^\natural = \frac{1}{\sqrt{1+B_0^2}}\Big(B_0 t + z\tan\varphi\Big)\,, \tag{4.35}$$

from which one may notice that $X^\natural$ is also a harmonic function satisfying $\nabla^2 X^\natural = 0$ in the ρ and φ space. This is actually a general result that is true of all superfunnels and super-multipole configurations.

5. Systematics

We now present a systematic analysis of 1/4 supersymmetric D2-branes in three Euclidean space dimensions. For this purpose it is convenient to revert to the cartesian space coordinates $X^a; a = 1,2,3$, and worldvolume coordinates (t, σ^1, σ^2) and fix only the time reparametrizations by the usual gauge choice $T = t$. We assume that the worldvolume fields $(X^a, \mathbf{E}, \mathcal{B})$ are t-independent.

Because of the restriction to a 3-dimensional space, we have

$$\Delta^2 = \sum_a \left(G_a G_a - H^a H^a\right) + \mathcal{B}^2. \tag{5.1}$$

where

$$G_a = \frac{1}{2}\epsilon_{abc} \boldsymbol{\nabla} X^b \times \boldsymbol{\nabla} X^c\,, \qquad H^a = \mathbf{E} \times \boldsymbol{\nabla} X^a\,. \tag{5.2}$$

In this notation, the supersymmetry preservation condition becomes

$$\left[H^a \Gamma_{a\natural} + \frac{1}{2}\epsilon^{abc} G_a \Gamma_T \Gamma_{bc} + \mathcal{B}\,\Gamma_{T\natural} - \Delta\right]\epsilon = 0\,. \tag{5.3}$$

Because the spacetime is effectively assumed to be 4-dimensional, the results we obtain must be the same as if we had started from the super-D2-brane in this spacetime dimension. Consequently, we may interpret ϵ as a complex 4-component Dirac spinor[2] and the matrices $(\Gamma_T, \Gamma_a; a = 1,2,3)$ as standard four-dimensional 4×4 Dirac matrices and $-i\Gamma_\natural$ as their product. A convenient representation is

$$\Gamma_T = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}, \qquad \Gamma_a = \begin{pmatrix} 0 & \sigma_a \\ \sigma_a & 0 \end{pmatrix}, \tag{5.4}$$

in which case

$$\Gamma_\natural = i\Gamma_T \Gamma_{123} = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}, \tag{5.5}$$

where $\mathbf{1}$ is a 2×2 unit matrix. The supersymmetry preservation condition is now

$$\begin{pmatrix} -\Delta & a_0 + \sum_a a_a \sigma_a \\ a_0 - \sum_a a_a \sigma_a & -\Delta \end{pmatrix} \begin{pmatrix} \epsilon_+ \\ \epsilon_- \end{pmatrix} = 0\,, \tag{5.6}$$

where $\epsilon_\pm$ are two-component SU(2)-spinors spanning the $\mp$ eigenspaces of $\Gamma_\natural$, and

$$a_0 = \mathcal{B}\,, \qquad a_a = H^a + iG_a\,. \tag{5.7}$$

Note the manifest SU(2) invariance of the equation (5.6), and its invariance under a common U(1) phase rotation of $\epsilon_\pm$. This equation is equivalent to

$$\begin{pmatrix} \epsilon_3 \\ \epsilon_4 \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} a_0 - a_3 & -a_1 + ia_2 \\ -a_1 - ia_2 & a_0 + a_3 \end{pmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}. \tag{5.8}$$

[2] Supersymmetry requires the spinor parameter ϵ to be a Dirac spinor. One way to see this is to observe that the four-dimensional super-D2-brane action is equivalent to the action of the 5-dimensional supermembrane in exactly the same way as the string theory super-D2-brane action is equivalent to the action of the 11-dimensional supermembrane.

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where (ϵ_1, ϵ_2) are the components of ϵ_+ and (ϵ_3, ϵ_4) are the components of ϵ_-. Observe that $\sum_a H^a G_a = 0$, and hence that

$$\Delta^2 = a_0^2 - \sum_a a_a a_a \,. \tag{5.9}$$

Given that (ϵ_1, ϵ_2) are arbitrary complex constants, we can show that (ϵ_3, ϵ_4) are also constants if and only if (a_0, a_1, a_2, a_3), and hence Δ, are constants. This is the case of 1/2 supersymmetry. Our results of section 3 are recovered by fixing the worldspace diffeomorphism invariance appropriately. Preservation of any fraction of supersymmetry less than 1/2 is possible only if ϵ_1,ϵ_2 and their complex conjugates are subject to linear relations. We may use symmetries of (5.8) to bring any such relations into a 'standard' form using the SU(2) rotational invariance and U(1) phase invariance. Given a single real linear relation between (ϵ_1, ϵ_2) and their complex conjugates, we may use the phase invariance to arrange for ϵ_2 to be real. This leads to configurations that preserve at least 3/8 supersymmetry. However, the conditions for ϵ_3 and ϵ_4 to be constant remain the same as they were before so all configurations preserving at least 3/8 supersymmetry actually preserve 1/2 supersymmetry.

We learn from this that to find configurations preserving less than 1/2 supersymmetry we need to impose at least two real linear relations on ϵ_1, ϵ_2 and their complex conjugates. Given two real relations we will find configurations preserving at least 1/4 supersymmetry (we say "at least" because the 1/2 supersymmetric configurations will be included). Given three real relations we will find configurations preserving at least 1/8 supersymmetry. Any two real linear relations are equivalent to one complex linear relation on ϵ_1, ϵ_2. We may now use the SU(2) symmetry to arrange for this relation to be $\epsilon_2 = 0$. In this case, (5.8) implies

$$\epsilon_3 = \Delta^{-1} (a_0 - a_3)\, \epsilon_1 \,, \qquad \epsilon_4 = -\Delta^{-1} (a_1 + i a_2)\, \epsilon_1 \,, \tag{5.10}$$

In the case that there is one more linear relation, which will now be a relation between the real and imaginary parts of ϵ_1 we can use the U(1) invariance to arrange for this relation to be $\mathcal{I}m\epsilon_1 = 0$, so that ϵ_1 is real. In this case (5.8) again implies (5.10) but with ϵ_1 now restricted to be real. However, irrespective of whether ϵ_1 is real or complex, constancy of (ϵ_3, ϵ_4) requires both $(a_1 + i a_2)/\Delta$ and $(a_0 - a_3)/\Delta$ to be constant. It follows that all configurations preserving at least 1/8 supersymmetry actually preserve at least 1/4 supersymmetry, and that the conditions for this are

$$\mathcal{B} - \mathbf{E} \times \boldsymbol{\nabla} X^3 + i \boldsymbol{\nabla} X^1 \times \boldsymbol{\nabla} X^2 \propto \Delta \,, \qquad \left(\mathbf{E} - \boldsymbol{\nabla} X^3\right) \times \left(\boldsymbol{\nabla} X^1 + i \boldsymbol{\nabla} X^2\right) \propto \Delta \,. \tag{5.11}$$

From the imaginary part of the first of these equations we see that

$$\boldsymbol{\nabla} X^1 \times \boldsymbol{\nabla} X^2 = \mathcal{A}\, \Delta \,, \tag{5.12}$$

for some *constant* $\mathcal{A}$, which determines the projection of an area element of the D2-brane onto the X^1-X^2 plane. We must now consider separately the $\mathcal{A} = 0$ and $\mathcal{A} \neq 0$ cases.

5.1 $\mathcal{A} = 0$

When $\mathcal{A} = 0$, it is convenient to revert to the cylindrical 3-space coordinates of (4.1) and the worldspace coordinate notation of (4.2). We may then fix the worldspace diffeomorphism invariance by the gauge choice $Z = z$ and $\Phi = \varphi$. Equivalently,

$$X^1 = R(z, \varphi) \cos\varphi \,, \qquad X^2 = R(z, \varphi) \sin\varphi \,, \qquad X^3 = z \tag{5.13}$$

We now have

$$0 = \boldsymbol{\nabla} X^1 \times \boldsymbol{\nabla} X^2 = R R_z \,. \tag{5.14}$$

In other words, the vanishing of $\mathcal{A}$ implies that $R_z = 0$ (since $R = 0$ would imply a collapsed D2-brane of zero area). This leads to

$$a_0 = \mathcal{B}\,, a_1 = E_z\left(R_\varphi \cos\varphi - R\sin\varphi\right) - i\left(R_\varphi \sin\varphi + R\cos\varphi\right)\,,$$
$$a_2 = E_z\left(R_\varphi \sin\varphi + R\cos\varphi\right) + i\left(R_\varphi \cos\varphi - R\sin\varphi\right)\,, a_3 = -E_\varphi\,, \tag{5.15}$$

and hence to

$$\Delta^2 = \left(\mathcal{B} + E_\varphi\right)\left(\mathcal{B} - E_\varphi\right) + \left(1 - E_z^2\right)\left(R^2 + R_\varphi^2\right)\,. \tag{5.16}$$

Recalling that the conditions for preservation of 1/4 supersymmetry are those in (5.11), we now find that they become

$$\mathcal{B} + E_\varphi = c\Delta\,, \qquad \left(E_z - 1\right) e^{i\varphi}\left(R_\varphi + iR\right) = w\Delta\,, \tag{5.17}$$

for real constant c and complex constant w. In addition, we must take into account the Bianchi identity

$$\partial_\varphi E_z = \partial_z E_\varphi \tag{5.18}$$

and the Gauss law constraint

$$\left(R^2 + R_\varphi^2\right)\partial_z\left(E_z/\Delta\right) + \partial_\varphi\left(E_\varphi/\Delta\right) = 0\,. \tag{5.19}$$

Let us first consider the cases for which w in (5.17) is non-zero. In this case the ratio of the real and imaginary parts of the second of the supersymmetry conditions (5.17) yields a differential equation for R that has the solution

$$\mathcal{I}m\left(\bar{w}e^{i\varphi}\right) R = R_0 \tag{5.20}$$

for some constant R_0. This is a line in the X^1-X^2 plane parametrized by φ, so the D2-brane geometry is planar. Further analysis shows that the BI fields are constant in local cartesian coordinates, so $w \neq 0$ leads back to the 1/2 supersymmetric planar D2-branes already discussed.

The interesting case is therefore $w = 0$, in which case supersymmetry requires $E_z = 1$, and hence

$$\Delta^2 = \left(\mathcal{B} - E_\varphi\right)\left(\mathcal{B} + E_\varphi\right) = c\left(\mathcal{B} - E_\varphi\right)\Delta\,. \tag{5.21}$$

This means that $\mathcal{B}$ and E_φ are both proportional to Δ and hence to each other. As $\mathcal{B}$ cannot vanish (for positivity of Δ^2), we may write

$$E_\varphi = \beta\,\mathcal{B}\,, \qquad \beta = \frac{c^2 - 1}{c^2 + 1}\,, \tag{5.22}$$

and hence

$$\Delta = \sqrt{1 - \beta^2}\,\mathcal{B}\,. \tag{5.23}$$

The remaining condition for supersymmetry ($\mathcal{B} + E_\varphi = c\Delta$) is now an identity. Since $E_\varphi \propto \Delta$, the Gauss law states that Δ is a function only of φ, which means that both E_φ and $\mathcal{B}$ are also functions only of φ, since they are proportional and their sum is proportional to Δ.

We have now found 1/4 supersymmetric D2-branes that are z-independent and hence invariant under translations along the Z-axis. In fact, we have recovered the supertubes ($\beta = 0$) and twisted supertubes ($\beta \neq 0$) of section 4.1. In that case we found that $E_z = \pm 1$ but the $E_z = -1$ case is obtained from the $E_z = 1$ case by a rotation, and in this section we effectively used the rotational invariance to arrange for $E_z = 1$ rather than $E_z = -1$.

5.2 $\mathcal{A} \neq 0$

When $\mathcal{A} \neq 0$, we may choose

$$X^1 = \sigma^1\,, \qquad X^2 = \sigma^2 \qquad X^3 = Z(\sigma^1, \sigma^2) \tag{5.24}$$

In this gauge, $\Delta = 1/|\mathcal{A}|$, and (5.11) is equivalent to

$$E = \boldsymbol{\nabla} Z + \mathbf{k}\,, \qquad \mathcal{B} = B_0 + \mathbf{k} \times \boldsymbol{\nabla} Z\,, \tag{5.25}$$

for constant B_0 and constant 2-vector $\mathbf{k}$. The worldspace metric is

$$h_{ij} = \delta_{ij} + \partial_i Z \partial_j Z\,, \tag{5.26}$$

from which one deduces, according to (2.26) and (2.27), the Gauss law

$$\nabla^2 Z = \boldsymbol{\nabla} Z \times \boldsymbol{\nabla} \left(\mathbf{k} \times \boldsymbol{\nabla} Z\right)\,, \tag{5.27}$$

so that Z is harmonic when $\mathbf{k} = \mathbf{0}$. To make contact with section 4 we define new coordinates (z, φ) by

$$\sigma^1 = R\cos\varphi\,, \qquad \sigma^2 = R\sin\varphi\,, \tag{5.28}$$

where the function $R(z, \varphi)$ is determined implicitly by the requirement that

$$Z(\sigma^1, \sigma^2) = z\,. \tag{5.29}$$

The Laplace equation for Z is now equivalent to the differential equation (4.24) for R, and the BI field-strength 2-form is

$$F = dz \wedge dt + B_0 R R_z dz \wedge d\varphi\,. \tag{5.30}$$

We have therefore recovered the superfunnels, and other 'supershapes' of section 4.

It remains to analyse the case of non-zero $\mathbf{k}$ or, equivalently, non-zero $k \equiv |\mathbf{k}|$. If the expressions (5.25) are used to compute Δ as given in (5.9) in terms of $\boldsymbol{\nabla} Z$, then one finds that Δ is a constant, as required, if and only if

$$B_0\, \mathbf{k} \times \boldsymbol{\nabla} Z - \mathbf{k} \cdot \boldsymbol{\nabla} Z = \ell \tag{5.31}$$

for some constant ℓ, which is not independent of those already introduced since

$$\Delta^2 = B_0^2 - \left(1 + k^2\right) + 2\ell\,. \tag{5.32}$$

The solution of this equation is

$$Z = W(z_+) + \frac{\ell}{k\sqrt{1 + B_0^2}}\, z_-\,, \tag{5.33}$$

where

$$\begin{aligned} z_+ &= \frac{1}{k\sqrt{1+B_0^2}} \left[\left(B_0 k_1 - k_2\right)\sigma^1 + \left(B_0 k_2 + k_1\right)\sigma^2\right]\,, \\ z_- &= \frac{1}{k\sqrt{1+B_0^2}} \left[-\left(B_0 k_2 + k_1\right)\sigma^1 + \left(B_0 k_1 - k_2\right)\sigma^2\right]\,. \end{aligned} \tag{5.34}$$

The Gauss law constraint (5.27) now yields

$$\left(1 + B_0^2 + \ell\right) W'' = 0\,. \tag{5.35}$$

The term in parentheses cannot vanish for real Δ, so we conclude that $W''=0$, and hence that W is a linear function of z_+. Thus,

$$Z = Z_0 + Cz_+ + \frac{\ell}{k\sqrt{1+B_0^2}}\, z_- \,, \tag{5.36}$$

where Z_0 and C are constants. The geometry is therefore planar. Moreover, the BI fields are constant, so that these configurations with $k \neq 0$ are actually just the 1/2 supersymmetric planar D2-branes that we already know about.

6. D2 in 4D

There is a further class of 1/4 supersymmetric configuration of D2-branes that is suggested by the well-known 1/4 supersymmetric Kähler calibrated M2-branes. The latter is a configuration that has two asymptotic planes and may therefore be interpreted as an intersection of two M2-branes, although the intersection is non-singular so the configuration can be found as a solution of the worldvolume equations of motion for a single supermembrane. It is obvious that dimensional reduction of such a configuration could lead (depending on its orientation) to a similar D2-brane configuration. The geometry is intrinsically 4-dimensional, which explains why we did not see it in our previous exhaustive analysis of D2-branes in 3D. As we now show, a novelty of the D2-brane version of this 'intersecting brane' configuration is the possibility, consistent with 1/4 supersymmetry, of a superposed uniform BI magnetic field.

As we expect the geometry to be 4-dimensional, we set $X^5 = X^6 = X^7 = X^8 = X^9 = 0$. We fix the worldspace parametrization invariance by setting

$$X^3 = \sigma^1\,, \qquad X^4 = \sigma^2\,, \tag{6.1}$$

leaving (X^1, X^2) as the surviving worldvolume scalar fields. We now find that

$$\det h = 1 + \left|\nabla X^1\right|^2 + \left|\nabla X^2\right|^2 + \left(\nabla X^1 \times \nabla X^2\right)^2 \,. \tag{6.2}$$

We will set the BI electric field to zero, so the Gauss law is satisfied trivially, and

$$\Delta^2 = \det h + \mathcal{B}^2 \,. \tag{6.3}$$

The kappa-symmetry matrix of (2.25) is now

$$\Gamma = \Delta^{-1}\left[\Gamma_{T12} + \Gamma_{T13}\partial_2 X^1 - \Gamma_{T24}\partial_1 X^2 - \Gamma_{T23}\partial_1 X^1 + \Gamma_{T14}\partial_2 X^2 + \Gamma_{T34}\left(\nabla X^1 \times \nabla X^2\right) + \Gamma_{T\natural}\mathcal{B}\right]. \tag{6.4}$$

Imposing the conditions

$$\Gamma_{1234}\epsilon = \epsilon\,, \tag{6.5}$$

and the Cauchy-Riemann (CR) equations

$$\partial_1 X^1 = -\partial_2 X^2\,, \qquad \partial_1 X^2 = \partial_2 X^1\,, \tag{6.6}$$

one finds that

$$\sqrt{\det h} = 1 + \frac{1}{2}\left|\nabla X^1\right|^2 + \frac{1}{2}\left|\nabla X^2\right|^2\,, \tag{6.7}$$

and the supersymmetry condition $\Gamma\epsilon = \epsilon$ reduces to

$$\left[\sqrt{\det h}\,\Gamma_{T12} + \mathcal{B}\,\Gamma_{T\natural}\right]\epsilon = \Delta\,\epsilon\,. \tag{6.8}$$

Provided that the magnetic field $\mathcal{B}$ is uniform, which means that

$$\mathcal{B} = B\sqrt{\det h}\,, \tag{6.9}$$

for some constant B, the two constraints (6.5) and (6.8) are compatible and imply preservation of 1/4 supersymmetry. The CR equations imply that the complex field $\mathcal{Z} = X^1 + iX^2$ is a holomorphic function of the complex worldspace coordinate $\zeta = \sigma^1 - i\sigma^2$. Equivalently, one has $f(\mathcal{Z},\zeta) = 0$ for holomorphic function f of the two complex variables $(\mathcal{Z},\zeta)$; this implies that the D2-brane is Kähler calibrated. The novelty here is the additional uniform magnetic field density.

7. General cross-sections

We know from [14] that the most general cross section of a supertube is a an arbitrary curve in the 8-dimensional space transverse to the axis of the tube. An obvious question is whether this result generalizes to twisted supertubes and superfunnels. We now aim to answer this question, using the cartesian coordinate approach of [14]. We first set

$$X^I = (Z, Y^A) \qquad (A = 1,\ldots,8)\,, \tag{7.1}$$

where the Z-axis will be the axis of the supertube or superfunnel. Then we observe that

$$\begin{aligned}\Delta^2 \,=\, & \Big[(\boldsymbol{\nabla} Z + \mathbf{E}) \times \boldsymbol{\nabla}\vec{Y}\Big] \cdot \Big[(\boldsymbol{\nabla} Z - \mathbf{E}) \times \boldsymbol{\nabla}\vec{Y}\Big] - (\mathbf{E} \times \boldsymbol{\nabla} Z)^2 \\ & + \sum_{A>B} \big(\boldsymbol{\nabla} Y^A \times \boldsymbol{\nabla} Y^B\big)\big(\boldsymbol{\nabla} Y^A \times \boldsymbol{\nabla} Y^B\big) + \mathcal{B}^2\,,\end{aligned} \tag{7.2}$$

where $\boldsymbol{\nabla} Y^A$ are the components of the 8-vector $\boldsymbol{\nabla}\vec{Y}$ and the dot product is the standard Euclidean inner product on $\mathbb{E}^8$. In the same notation we have

$$\begin{aligned}\Gamma \,=\, \Delta^{-1}\Big[& \big(\boldsymbol{\nabla} Z \times \boldsymbol{\nabla} Y^A\big)\,\Gamma_{TZA} + \frac{1}{2}\big(\boldsymbol{\nabla} Y^A \times \boldsymbol{\nabla} Y^B\big)\,\Gamma_{TAB} \\ & + (\mathbf{E} \times \boldsymbol{\nabla} Z)\,\Gamma_{Z\natural} + \big(\mathbf{E} \times \boldsymbol{\nabla} Y^A\big)\,\Gamma_{A\natural} + \mathcal{B}\Gamma_{T\natural}\Big].\end{aligned} \tag{7.3}$$

For either superfunnels or supertubes, twisted or otherwise, we impose the constraint

$$\Gamma_{TZ\natural}\epsilon = \mp\epsilon\,, \tag{7.4}$$

which allows us to rewrite (7.3) as

$$\Gamma = \Delta^{-1}\Big[(\mathbf{E} \mp \boldsymbol{\nabla} Z) \times \boldsymbol{\nabla} Y^A\,\Gamma_{A\natural} + (\mathbf{E} \times \boldsymbol{\nabla} Z)\,\Gamma_{Z\natural} \;+\; \frac{1}{2}\big(\boldsymbol{\nabla} Y^A \times \boldsymbol{\nabla} Y^B\big)\,\Gamma_{TAB} + \mathcal{B}\Gamma_{T\natural}\Big]. \tag{7.5}$$

To proceed, we solve the Bianchi identity (2.22) by setting

$$\mathbf{E} = \boldsymbol{\nabla} V \tag{7.6}$$

for some electric potential function $V(x,\varphi)$. As this function depends on E_φ, we consider first the cases in which $E_\varphi = 0$, which are the superfunnels and untwisted supertubes.

7.1 Superfunnels

Let us choose

$$V = \pm Z \tag{7.7}$$

In this case the supersymmetry preservation condition becomes

$$\left[\frac{1}{2}\left(\boldsymbol{\nabla} Y^A \times \boldsymbol{\nabla} Y^B\right)\Gamma_T\Gamma_{AB} + \mathcal{B}\Gamma_{T\natural} - \Delta\right]\epsilon = 0\,, \tag{7.8}$$

where now

$$\Delta^2 = \sum_{A>B}\left(\boldsymbol{\nabla} Y^A \times \boldsymbol{\nabla} Y^B\right)\left(\boldsymbol{\nabla} Y^A \times \boldsymbol{\nabla} Y^B\right) + \mathcal{B}^2\,. \tag{7.9}$$

If there are to be no further constraints on ϵ, we must have

$$\boldsymbol{\nabla} Y^A \times \boldsymbol{\nabla} Y^B = \Delta\,\Omega^{AB} \tag{7.10}$$

for some *constant* antisymmetric 8×8 matrix Ω. One possibility is $\Omega = 0$, which arises when $\partial_x \vec{Y} = 0$. This leads to the standard (untwisted) supertube with arbitrary cross-section. We skip the details since the untwisted supertube may be considered as a special (zero-twist) case of the twisted supertube, to be discussed in the following subsection. However, for purposes of comparison with the superfunnel, we observe that when $\partial_x \vec{Y} = 0$ the equation (7.10) places no restriction on $\partial_\varphi \vec{Y}$, and this is what allows the supertube cross-section to be an arbitrary curve in the transverse 8-space. As we shall now see, the situation is quite different when Ω is non-zero.

Given that Ω is non-zero, (7.10) implies (i) that Δ is a constant in the gauge

$$Y^1 = \sigma^1\,, \qquad Y^2 = \sigma^2\,, \tag{7.11}$$

and (ii) that the projection of the D2-brane geometry on the 8-dimensional Euclidean space with coordinates Y is a plane, which we may orient such that the only non-zero entries of Ω are $\Omega^{12} = -\Omega^{21} = 1/\Delta$. We then have $\Delta^2 = 1 + \mathcal{B}^2$ but since Δ is a constant, we have $\mathcal{B} = B$ for some constant B. The supersymmetry preservation condition (7.8) now becomes

$$\Gamma_T\left[\Gamma_{12} + B\Gamma_\natural\right]\epsilon = \Delta\epsilon\,, \qquad \Delta = \sqrt{1+B^2}\,. \tag{7.12}$$

By itself, this condition would imply preservation of 1/2 supersymmetry, as would (7.4). These two constraints on ϵ are compatible because $\Gamma_{TZ\natural}$ commutes with both $\Gamma_T\Gamma_{12}$ and $\Gamma_{T\natural}$, and the two together imply preservation of 1/4 supersymmetry.

To summarize, we have now shown that any configuration with constant $\mathcal{B}$, and electric field $\mathbf{E} = \pm\boldsymbol{\nabla} Z$, preserves 1/4 supersymmetry. However, we have still to impose the Gauss law constraint. Since $\mathcal{D}_i = \pm\Delta^{-1}\partial_i Z$ and Δ is constant, the Gauss law is

$$\nabla^2 Z = 0\,. \tag{7.13}$$

We have now recovered our earlier result for superfunnels, but now we have seen that the superfunnel cross section is necessarily planar.

7.2 Twisted supertubes

For convenience of comparison with our analysis of section 4, we make the notational change $(\sigma^1, \sigma^2) = (z, \varphi)$. Returning to (7.5) we now partially fix the worldspace diffeomorphisms by the gauge choice

$$Z = z\,, \tag{7.14}$$

and we further assume that all other worldvolume fields are z-independent. We then have a D2-brane configuration that is invariant under translations along the Z-axis. Also, (7.5) simplifies to

$$\Gamma = \Delta^{-1}\left[\partial_\varphi Y^A \Gamma_{ATZ} + E_z \partial_\varphi Y^A \Gamma_{A\natural} - E_\varphi \Gamma_{Z\natural} + \mathcal{B}\Gamma_{T\natural}\right], \tag{7.15}$$

where

$$\Delta^2 = \left(1 - E_z^2\right) |\partial_\varphi \vec{Y}|^2 - E_\varphi^2 + \mathcal{B}^2 . \tag{7.16}$$

The twisted supertubes are now found by setting

$$E_z = \pm 1 , \qquad \Gamma_{TZ\natural}\epsilon = \mp\epsilon . \tag{7.17}$$

The Gauss law is now an identity, as is the Bianchi identity, and the supersymmetry preservation condition is

$$\left(\mathcal{B}\Gamma_{T\natural} - E_\varphi \Gamma_{Z\natural}\right)\epsilon = \Delta\epsilon, \qquad \Delta^2 = \mathcal{B}^2 - E_\varphi^2 . \tag{7.18}$$

For this to be satisfied without further constraint on ϵ, we require that

$$E_\varphi = \beta\mathcal{B} , \qquad \beta^2 < 1 , \tag{7.19}$$

for some constant β, as we found in (4.19). Given the above electric field components, we see that the electric potential is

$$V(z,\varphi) = \pm z + \beta \int^\varphi d\varphi' \mathcal{B}(\varphi') . \tag{7.20}$$

As expected, this coincides with the choice $V = \pm Z$ of the previous subsection when $\beta = 0$ (since we are now working in the gauge $Z = z$). The supersymmetry preservation condition is now

$$\frac{1}{\sqrt{1-\beta^2}}\left(\Gamma_T - \beta\Gamma_Z\right)\Gamma_\natural\,\epsilon = \epsilon . \tag{7.21}$$

As both $\Gamma_{T\natural}$ and $\Gamma_{Z\natural}$ commute with $\Gamma_{TZ\natural}$, this constraint is compatible with (7.4) and the two together imply preservation of 1/4 supersymmetry.

The above result can be summarized as follows. The static D2-brane configuration with

$$Z = z , \qquad \vec{Y} = \vec{Y}(\varphi) , \tag{7.22}$$

and

$$F = \pm dt \wedge dz + q\mathcal{B}(\varphi)\, dt \wedge d\varphi + \mathcal{B}(\varphi)\, dz \wedge d\varphi , \tag{7.23}$$

preserves 1/4 supersymmetry for arbitrary functions $\vec{Y}(\varphi)$, and arbitrary positive function $\mathcal{B}(\varphi)$. Although it is not necessary, we may suppose that φ is periodically identified, such that $\varphi \sim \varphi + 2\pi$ without loss of generality, and in this case we have a tubular configuration, translationally invariant along the Z-axis, and with a cross-section determined by an arbitrary closed curve in the transverse 8-dimensional space. For $\beta = 0$ this is the general D2-brane supertube of [14], for which the electric field lines are parallel to the Z-axis. Note that although the function $\mathcal{B}(\varphi)$ is arbitrary, we have still to fix the φ-reparametrization invariance. As $\mathcal{B} > 0$, this can be done by setting $\mathcal{B} = B$ for some positive constant B. Since $B = \oint d\varphi \mathcal{B}(\varphi)$, which is reparametrization invariant, different choices of B represent distinct configurations.

When $\beta \neq 0$, we have a new class of 1/4 supersymmetric D2-branes for which the electric field lines are at a non-zero angle to the Z-axis and therefore twist around it. These could be called "twisted supertubes" but they are actually just standard supertubes boosted in the Z direction. Note that $\Delta = \mathcal{B}\sqrt{1-\beta^2}$ is boost invariant, so a boost from rest increases $\mathcal{B}$ and hence the energy density. In fact, the energy density is

$$\mathcal{H} = \frac{|\partial_\varphi \vec{Y}|^2 + B^2}{B\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}}\left[|\mathcal{D}|_{\beta=0} + B\right] , \tag{7.24}$$

as expected for a boost of the supertube with velocity β.

8. Discussion

The work reported here began with the aim of clarifying some aspects of the supersymmetry preservation condition for solutions of the Dirac-Born-Infeld equations for a D2-brane in a 3-dimensional Euclidean space. We have presented an exhaustive analysis of the conditions for supersymmetry preservation under this restriction and one result of this analysis is that all supersymmetric D2-branes of this type preserve either 1/2 or 1/4 supersymmetry.

The general 1/2 supersymmetric solution includes the obvious planar D2-brane vacuum but also includes other planar solutions with constant Born-Infeld fields. All these planar D2-branes lift to a planar M2-brane in M-theory, but the identification of the M-theory circle coordinate implies a finer classification of 1/2 supersymmetric D2-branes than one has for planar M2-branes in the 11-dimensional Minkowski vacuum. This classification arises by considering the velocity of the intersection of the M2-brane with what we have called an 'ether'-brane, and a 'null intersection' leads to a 'dyonic' D2-brane.

In the case of 1/4 supersymmetry, we found a new class of 'twisted' supertubes, with electric field lines that twist around the tube. These can be understood as supertubes boosted along the direction of translational invariance. The boost invariance in this direction is broken by the magnetic field density, which is why a boost generates a new solution. We also found 'superfunnels', which are tubular configurations with arbitrary planar cross section, with a scale that varies (exponentially, on average) along the tube. Various other 1/4 supersymmetric 'supershapes' were found to be possible, including asymptotically planar D2-branes.

We have also analysed the conditions for 1/2 and 1/4 supersymmetry without the restriction to an embedding in 3-dimensional space. In four space dimensions there are Kähler-calibrated minimal surfaces that are 1/4 supersymmetric solutions of the DBI equations with vanishing BI fields, and we found a generalization of this that allows for a uniform magnetic field. Our analysis was not exhaustive so there may be other types of 1/4 supersymmetric time-independent solutions for which the embedding space has more than three dimensions, but we suspect that nothing essentially new is possible.

What is known is that supertubes may have a cross-section that is an arbitrary curve in the 8-dimensional space transverse to the tube, and we have shown that the same applies to twisted supertubes. In contrast, the cross-section of a superfunnel was found to be necessarily planar. This difference might seem surprising but the translational invariance of supertubes makes possible string-theory dual configurations in which the arbitrary cross-section becomes an arbitrary wave profile, and this 'explanation' is not available to superfunnels.

It would be interesting to extend the considerations of this paper to lower fractions of supersymmetry, and also to extend the analysis to Dp-branes with $p > 2$. For example, for $D5$-branes one expects the BPS conditions classified in [17] for gauge fields in a flat six-dimensional spacetime to be relevant. We leave this to future investigations.

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